Reusability of FEM Software: A Program Family Approach

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Outline

• Finite Element Method (FEM)
• Program Family
• A FEM Program Family: FEMBA
  - Goal-Oriented Commonality Analysis (CA)
  - Documentation
  - Code Generation
Finite Element Method

• A numerical technique for solving PDEs

• A website maintained by Roger Young and Ian MacPhedran lists more than 100 public domain FEM programs

Maximum Principal Stress in a Paraglider
Solved by CalculiX
Program Family

• A set of programs that share a common set of features

• A commonality: the protocol

• A variability: the number of features

Pictures from CrunchGear
A FEM Program Family: FEMBA

FEMBA simulates a beam under an applied load using FEM
Goal-Oriented CA

Goals

Theoretical Models

Theoretical Assumptions

Computational Models

Computational Assumptions

Commonalities

Variabilities
Examples of CA for FEMBA (1)

- **G1 (gDis)**: FEMBA can solve for the displacement of a beam;
- **TA7**: The weight of the beam is neglected;
- **TM1 (tmDis)**: The equation to be solved for displacement is \( \frac{d^4w}{dx^4} = \frac{p(x)}{EI} \);
- **CA1 (caFEM)**: The problems are solved using FEM;
- **CM1 (cmDis)**: The formula for solving the displacement of each node is \( Ka = F \);
Examples of CA for FEMBA (2)

- **C4** (cDOF): Each node has two degrees of freedom (DOF)
- **VP1** (vpNumNodes): Each element has different number of nodes
- **D5** (dStiff): \( \mathbf{K} = \int \mathbf{B}^T \mathbf{E} \mathbf{I} \mathbf{B} \, dx \)
Documentation

• A program usually needs to be changed to be reused
• Documentation of variabilities and traceability matrices can help with the changes
Documentation: Variabilities

VP1
vpNumNodes
[2 .. MNN]
V1(n)
vNN(n)

VP2
vpNumBNodes
[2 .. MNBN]
V2(n)
vNBN(n)

VP3
vpIntAlg
V3_1
vDirect
V3_2
vGaussQuad

VP4
vpNumIntPts
[2 .. MIP]
V4(n)
vNIP(n)

Requires_v_vp
## Documentation: Traceability Matrix

Traceability Matrix between Requirements and Modules (partial)

<table>
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<tr>
<th></th>
<th>mInMesh</th>
<th>mOutput</th>
<th>mControl</th>
<th>mStiff</th>
<th>mLoad</th>
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</table>
1. Find the shape functions
   \( \phi_i(s) = c_{i1} + c_{i2}s + c_{i3}s^2 + c_{i4}s^3 \) for
   \( w = w_1 \phi_1 + L \theta_1 \phi_2 + w_2 \phi_3 + L \theta_2 \phi_4 \)
   \( \theta = w' \)

   \[
   \begin{align*}
   \varphi_1(0) &= 1 & \varphi_1'(0) &= 0 & \varphi_1(1) &= 0 & \varphi_1'(1) &= 0 \\
   \varphi_2(0) &= 0 & \varphi_2'(0) &= 1 & \varphi_2(1) &= 0 & \varphi_2'(1) &= 0 \\
   \varphi_3(0) &= 0 & \varphi_3'(0) &= 0 & \varphi_3(1) &= 1 & \varphi_3'(1) &= 0 \\
   \varphi_4(0) &= 0 & \varphi_4'(0) &= 0 & \varphi_4(1) &= 0 & \varphi_4'(1) &= 1 \\
   \end{align*}
   \]
2. Compute kinematics matrix $\mathbf{B} = A \mathbf{N}$, where $A$ is the operator $(d^2/ds^2)/L^2$ and $\mathbf{N} = [\phi_1 \ L\phi_2 \ \phi_3 \ L\phi_4]$

3. Compute stiffness matrix $\mathbf{K} = \int (\mathbf{B}^T \mathbf{E} \mathbf{I} \mathbf{B}) \, dx$
Code Generation for FEMBA

• Calculating K is time consuming
• Use Maple to generate code for variabilities
• Use information hiding to develop FEMBA
Conclusion

• Program family approach can improve reusability by reusing commonalities
• Goal-oriented CA, documentation and code generation can improve reusability by helping with changes
Thank You!